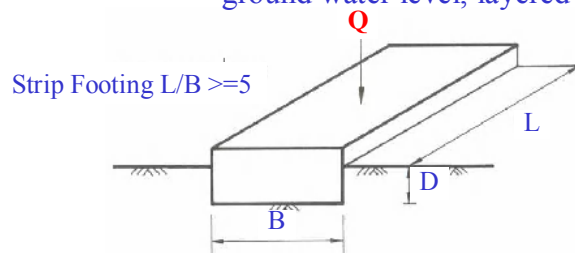


## 4. Shallow Foundation (浅基礎)

- **Foundation:** rigid footing (剛) (ex) mat F., raft F., spread F.  
flexible uniform loading (柔), (ex) embankment, tank
- **2D and 3D:** 2D strip footing ( $L \gg B$ )  
3D rectangular, circular footing
- **Loading condition:** central vertical loading  
eccentric vertical loading (偏心荷重 => モーメント)  
inclined loading (傾斜荷重)
- **Embedment (根入):**  $D < B$
- **Ground condition:** sandy soil ( $\phi'$ ), clayey soil ( $\phi_u = 0$ ), compressibility  
ground water level, layered soil,



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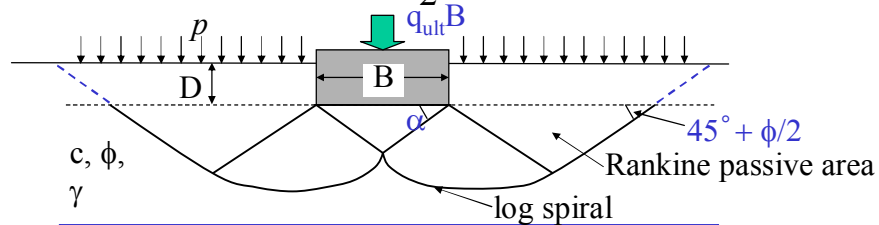
1

## 4.1 Bearing capacity

### 4.1.1 Bearing capacity equation

Terzaghi originally introduced bearing capacity equation and gave the **bearing capacity factors** ( $N_c, N_q, N_\gamma$ ) using limit equilibrium method for 2D under center vertical loading.

$$q_{ult} = Q_{ult} / B = cN_c + q_s N_q + \frac{\gamma B}{2} N_\gamma \quad q_s = \gamma D + p \quad (1)$$



Derivation of  $q_{ult}$ : superimposing  $q_{ult} = q_{ult1} + q_{ult2}$   
 $q_{ult1}$ : bearing capacity of soil with  $c, q_s \neq 0, \gamma = 0$   
 $q_{ult2}$ : bearing capacity of soil with  $\gamma \neq 0, c = q_s = 0$

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## Terzaghi bearing capacity factors: $\alpha=\phi$

for rough base:

$$\left. \begin{aligned} N_q &= \frac{1}{1 - \sin \phi} \exp\left\{\left(\frac{3}{2}\pi - \phi\right) \tan \phi\right\} \\ N_c &= (N_q - 1) \cot \phi \\ N_\gamma &\approx (N_q - 1) \tan(1.4\phi) \end{aligned} \right\} (2)$$

for smooth base:

(Table 3.1, p158, Das text book)

$$\left. \begin{aligned} N_{q0} &= \frac{1 + \sin \phi}{1 - \sin \phi} \exp(\pi \tan \phi) \\ N_{c0} &= (N_{q0} - 1) \cot \phi \\ N_{\gamma 0} &= \frac{1}{2} N_\gamma = \frac{1}{2} (N_{q0} - 1) \tan(1.4\phi) \end{aligned} \right\} (3)$$

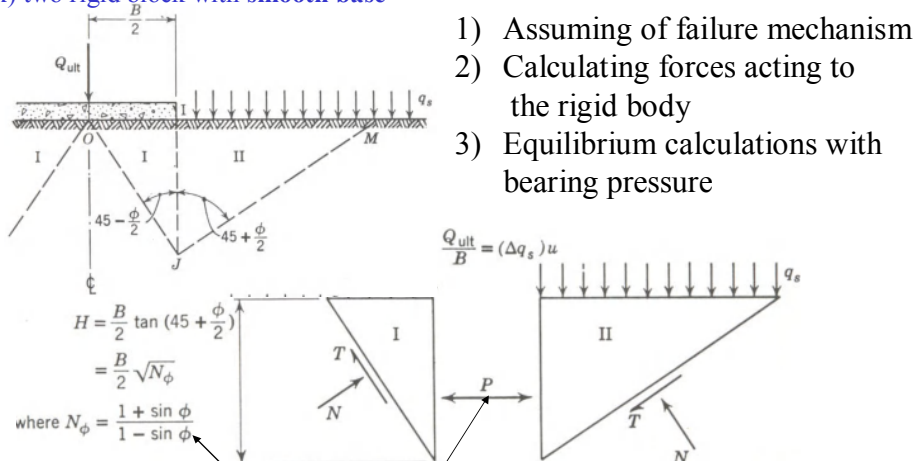
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## Examples of derivation of bearing capacity factors using limit equilibrium method

ex) two rigid block with smooth base



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## Equilibrium at the interface between two blocks

### Maximum force P that can be applied to passive wedge II

from Rankine's passive pressure equation

$$P = q_s H N_\phi + \frac{1}{2} \gamma H^2 N_\phi \quad K_p$$

$$P = q_s \frac{B}{2} N_\phi^{3/2} + \frac{1}{8} \gamma B^2 N_\phi^2 \quad (4)$$

### Maximum average pressure $Q_{ult}/B$ that can be applied to active wedge I

$$P = \frac{Q_{ult}}{B} \frac{H}{N_\phi} + \frac{1}{2} \gamma H^2 \frac{1}{N_\phi}$$

$$\frac{Q_{ult}}{B} = \frac{P}{H} N_\phi - \frac{1}{2} \gamma H = \left( \frac{2P}{B} - \frac{1}{4} \gamma B \right) \sqrt{N_\phi} \quad (5)$$

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(4) => (5)

$$\frac{Q_{ult}}{B} = q_s N_\phi^2 + \frac{1}{4} \gamma B N_\phi^{5/2} - \frac{1}{4} \gamma B N_\phi^{1/2}$$

$$\frac{Q_{ult}}{B} = q_s N_\phi^2 + \frac{\gamma B}{4} (N_\phi^{5/2} - N_\phi^{1/2}) \quad (6)$$

$$\left. \begin{aligned} N_q &= N_\phi^2 = K_p^2 \\ N_\gamma &= (N_\phi^{5/2} - N_\phi^{1/2}) / 2 \end{aligned} \right\} (7)$$

$$N_\phi = K_p = \frac{1 + \sin \phi}{1 - \sin \phi} = \tan^2 \left( 45^\circ + \frac{\phi}{2} \right) \quad (8)$$

$N_q$  and  $N_\gamma$  in eq.(7) are smaller than value derived by other method.  
compare eqs. (3) and (7)

ex) for  $\phi=30^\circ$   $N_{q(3)}=18$ ,  $N_{q(7)}=9$ ;  
for  $\phi=40^\circ$   $N_{q(3)}=64$ ,  $N_{q(7)}=21$

**Why??**

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## Bearing capacity factors: $\alpha=45^\circ+\phi/2$

for rough base:

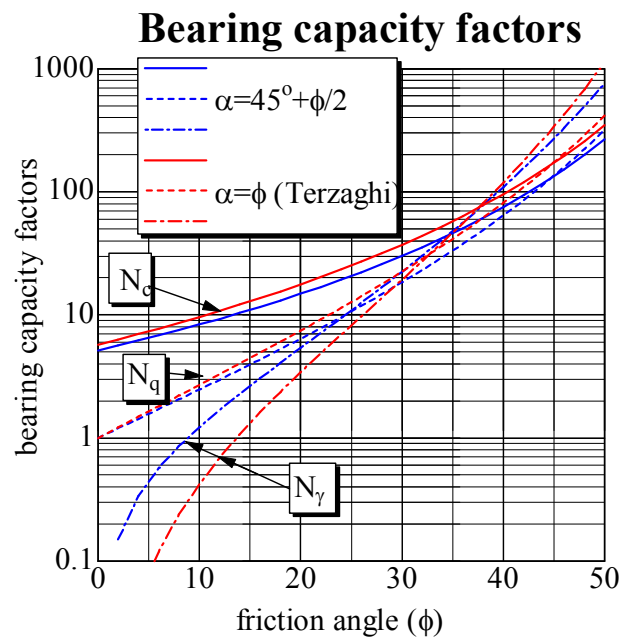
$$\left. \begin{aligned} N_q &= \tan^2\left(45^\circ + \frac{\phi}{2}\right) \exp(\pi \tan \phi) \\ N_c &= (N_q - 1) \cot \phi \\ N_\gamma &\approx 2(N_q + 1) \tan \phi \end{aligned} \right\} (9)$$

$N_q$ ,  $N_c$ ,  $N_\gamma$  are given by tabulated form or chart.  
(e.g., Das "Principle of Foundation Eng.")

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## 4.1.2 Effect of footing shape, eccentricity, inclined load, embedment

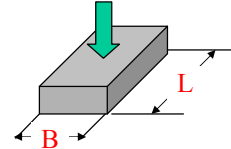
### General bearing capacity equation

$$q_{ult} = cN_c F_{cs} F_{cd} F_{ci} + q_s N_q F_{qs} F_{qd} F_{qi} + \frac{\gamma B}{2} N_\gamma F_{\gamma s} F_{\gamma d} F_{\gamma i} \quad (10)$$

$F_{cs}, F_{qs}, F_{\gamma s}$ : shape factors (形状係数) **3D effects**

$F_{cd}, F_{qd}, F_{\gamma d}$ : depth factors (深さ係数)

$F_{ci}, F_{qi}, F_{\gamma i}$ : inclination factors (傾斜荷重係数)



<b>Shape factors:</b> <i>semi-empirical</i>	Meyerhof(1963)	$F_{cs} = 1 + 0.2N_\phi \frac{B}{L} \quad 3D > 2D$	} (11)	Hansen(1970)	$F_{cs} = 1 + \frac{B N_q}{L N_c}$
		$F_{qs} = 1.0 \quad (\phi = 0^\circ)$		$F_{qs} = 1 + \frac{B}{L} \tan \phi$	
		$F_{qs} = F_{\gamma s} = 1 + 0.1N_\phi \frac{B}{L} \quad (\phi > 10^\circ)$		$F_{\gamma s} = 1 + 0.4 \frac{B}{L} \quad 3D < 2D$	
		$N_\phi = \tan^2 \left( \frac{1}{4} \pi + \frac{1}{2} \phi \right)$		for circular footing: $B/L=1$	

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### Theoretical values of shape factors

Bearing capacity of circular footing ( $B/L \sim 1$ ) can be solved by slip line method using cylindrical coordinate.

#### About $F_{cs}$

$q_{ult}$  of circular footing on  $\phi_u = 0$  material:  $6.05c_u \Rightarrow F_{cs} = 6.06/5.14 = 1.18$

*good agreement*

using eqs.(11) and (12) with  $\phi_u = 0$  and  $B/L = 1$ ,  $F_{cs} = 1.2$  and  $1.19$

#### About $F_{\gamma s}$

Slip line method gives larger  $N_\gamma$  for circular F. than strip F. for the same  $\phi$  value. That means  $F_{\gamma s} > 1$ , which is consistent with eq.(11) and inconsistent with eq.(12).



**Key words to explain  $N_\gamma$ :** stress dependency of  $\phi'$ , strain constraint (or  $\sigma'_2$ ) effect on  $\phi'$  and progressive failure or local failure.

## Slip Line Method for axisymmetric condition

two stress equilibrium equations with cylindrical coordinate

$$\frac{\partial \sigma_r}{\partial r} + \frac{\partial \tau_{rz}}{\partial z} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \quad \text{four unknown values}$$

$$\frac{\partial \tau_{rz}}{\partial r} + \frac{\partial \sigma_z}{\partial z} + \frac{\tau_{rz}}{r} = \gamma$$

three equations

failure criteria (Mohr-Coulomb)

$$\sigma_1 - \sigma_3 = c_u \quad \text{for undrained conditions } (\phi_u = 0)$$

$$\sigma'_1 - \sigma'_3 = 2c' \cos \phi' + (\sigma'_1 + \sigma'_3) \sin \phi' \quad \text{for drained conditions}$$

need one more assumption:  $\sigma_0 = \sigma_1$  or  $\sigma_3$  : Haar & Karman's assumption  
As a common assumption,  $\sigma_0 = \sigma_3$  is the most reliable

geometric condition

$$+ \begin{cases} \alpha(s_1) \text{ line: } \frac{dr}{dz} = \tan(\eta - 45^\circ + \frac{\phi'}{2}) \\ \beta(s_2) \text{ line: } \frac{dr}{dz} = \tan(\eta + 45^\circ - \frac{\phi'}{2}) \end{cases}$$

two differential eqs. expressing  $s$  and  $\eta$  along each slip line in terms of  $s$ , rotation of  $\eta$  and position of  $(r, z)$ .

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### $N_\gamma$ obtained from slip line methods

Bolton et al. (1991),  
Can. Geotech. Vol.30,  
p.1024-1033.

for same  $\phi$  value

$N_\gamma$  of strip (2D)

$N_\gamma$  of circular (3D)

$$F_{\gamma s} > 1$$

consistent with eq.(11)  
inconsistent with eq.(12)

$$\phi'_{\text{plane}} \sim 1.1 \phi'_{\text{triaxial}}$$

$\phi$	$\tan \phi$	$N_\gamma$			
		smooth base		rough base	
		strip	circular	strip	circular
5	0.09	0.09	0.06	0.62	0.68
10	0.18	0.29	0.21	1.71	1.37
15	0.27	0.71	0.60	3.17	2.83
20	0.36	1.60	1.30	5.97	6.04
25	0.47	3.51	3.00	11.6	13.5
30	0.58	7.74	7.10	23.6	31.9
31	0.60	9.1	8.6	27.4	38.3
32	0.62	10.7	10.3	31.8	46.1
33	0.65	12.7	12.4	37.1	55.7
34	0.67	15.0	15.2	43.5	67.6
35	0.70	17.8	18.2	51.0	82.4
36	0.73	21	22	60	101
37	0.75	25	27	71	124
38	0.78	30	33	85	153
39	0.81	36	40	101	190
40	0.84	44	51	121	238
41	0.87	53	62	145	299
42	0.90	65	78	176	379
43	0.93	79	99	214	480
44	0.97	97	125	262	619
45	1.00	120	160	324	803
46	1.04	150	210	402	1052
47	1.07	188	272	505	1384
48	1.11	237	353	638	1847
49	1.15	302	476	815	2491
50	1.19	389	621	1052	3403

$$N_{\gamma c} / N_{\gamma p} = F_{\gamma s}$$

0.86

0.84

$$F_{\gamma s} < 1$$

consistent with eq.(12)

at small pressure: difference of  $\phi$  is more:  $\Rightarrow$  smaller  $F_{\gamma s}$  from *small scale test*

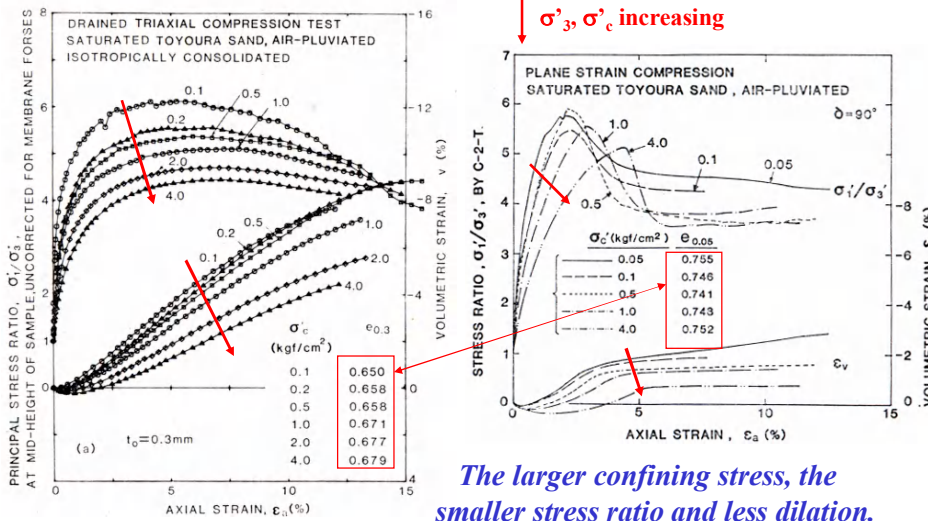
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# Stress dependency of stress-strain relations

Tatsuoka et al., S&F, Vo.26, No.1, p.65-84, 1984



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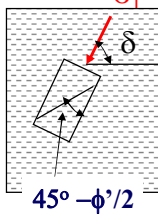
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# Relation ship between $\phi'$ and $\sigma'_3$

Tatsuoka et al., S&F, Vo.26, No.1, p.65-84, 1984

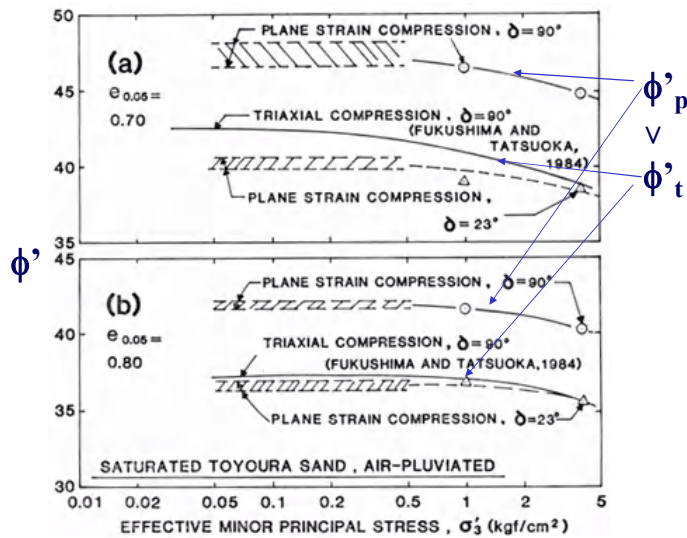
anisotropy of  $\phi'$



Strain constraint effect leads

$\sigma_2 > \sigma_3$   
in plane strain;  
 $\sigma_2 = \sigma_3$   
in triaxial comp.

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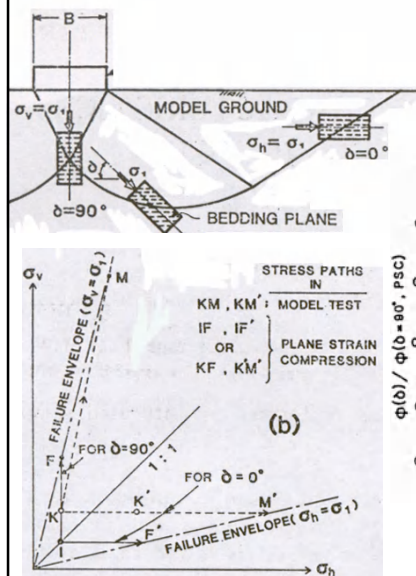


stress dependency of  $\phi'$

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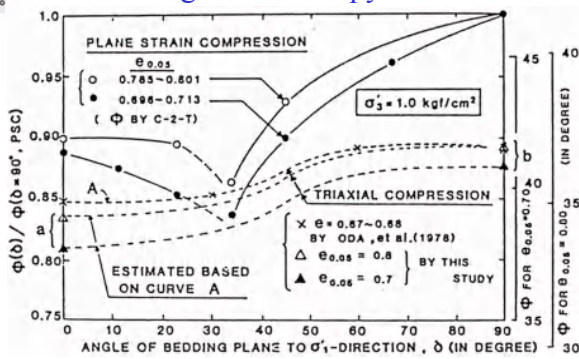
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### Anisotropy of $\phi'$



The denser, the higher anisotropy and the larger difference betw. P and T.

PS is higher anisotropy than TA.



Tatsuoka et al., S&F, Vo.26, No.1, p.65-84, 1984

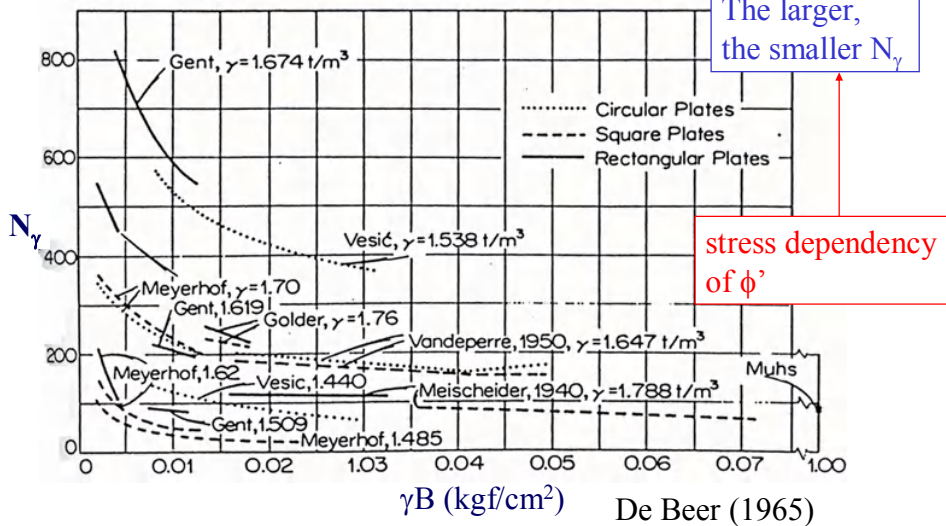
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### Scale effects of $N_\gamma$

-Effect of size of surface footing in sand-



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## Interpretation of the difference between eq.(11) and (12)

$F_{\gamma s}$  in eq. (12) was obtained from the model test, where mobilized  $\phi'$  value is higher for the sand under strip footings than circular footings.

$F_{\gamma s}$  in eq. (11) can be applied in the assumption that the mobilized  $\phi'$  values are the same for strip footings and circular footings.

“Which equation is better?? “ depends on the  $\phi'$  value used in the analysis,  $\phi'_p$  or  $\phi'_t$ ? If used  $\phi'$  is  $\phi'_p$ , eq.(12) is better and if  $\phi'_t$ , eq.(11) might be better.

As in actual design practice, the  $\phi'$  values are often estimated from empirical formulation (e.g., using N value) and not so accurate.

For safety reason, eq.(12) is normally used as the scale factor of  $N_{\gamma}$ .

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## Depth factors

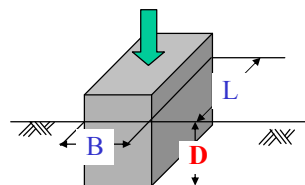
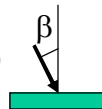
Hansen(1970)

$$\left. \begin{array}{l} \frac{D}{B} < 1 \\ F_{cd} = 1 + 0.4 \frac{D}{B} \\ F_{qd} = 1 + 2 \tan \phi (1 - \sin \phi)^2 \frac{D}{B} \\ F_{\gamma d} = 1 \end{array} \right\} (13) \quad \left. \begin{array}{l} \frac{D}{B} > 1 \\ F_{cd} = 1 + 0.4 \tan^{-1} \left( \frac{D}{B} \right) \text{ in radius} \\ F_{qd} = 1 + 2 \tan \phi (1 - \sin \phi)^2 \tan^{-1} \left( \frac{D}{B} \right) \\ F_{\gamma d} = 1 \end{array} \right\} (14)$$

## Inclination factors

Meyerhof(1963)

$$\left. \begin{array}{l} F_{ci} = F_{qi} = \left( 1 - \frac{\beta^\circ}{90^\circ} \right)^2 \\ F_{\gamma i} = \left( 1 - \frac{\beta}{\phi} \right)^2 \end{array} \right\} (15)$$



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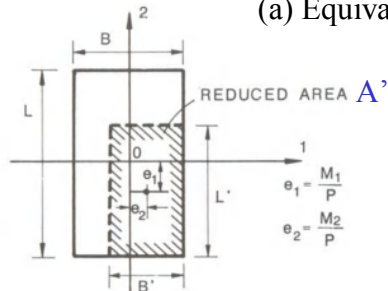
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# Reduced footing area for eccentric load

Foundation Engineering (Fang)

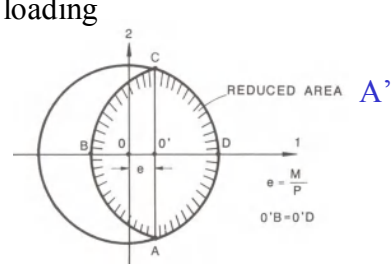


(a) Equivalent loading



$$L \Rightarrow L' = L - 2e_1, \quad B \Rightarrow B' = B - 2e_2 \quad (16)$$

(b) Rectangular footing



$$A' = 2S = B'L', \quad S = \frac{\pi R^2}{2} \left[ e \sqrt{R^2 - e^2} + R^2 \arcsin(e/R) \right]$$

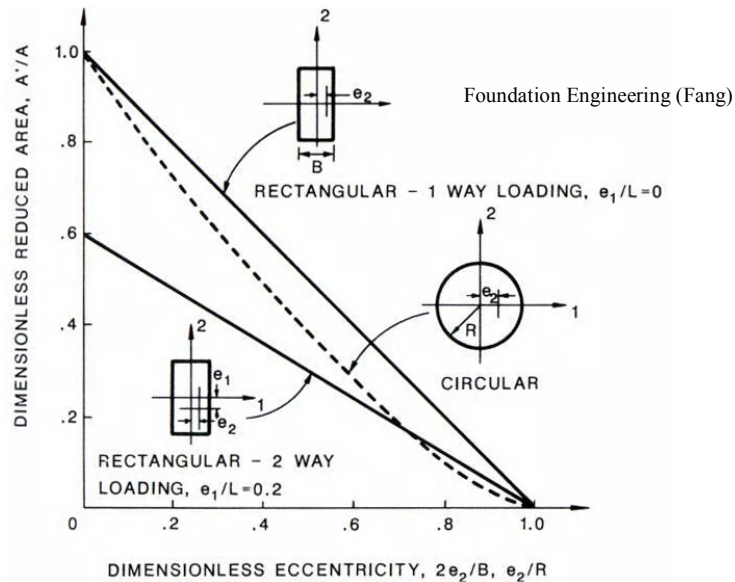
$$L' = \left[ 2S \left( \frac{R+e}{R-e} \right)^{0.5} \right]^{0.5}, \quad B' = L' \left( \frac{R+e}{R-e} \right)^{0.5} \quad (17)$$

(c) Circular footing

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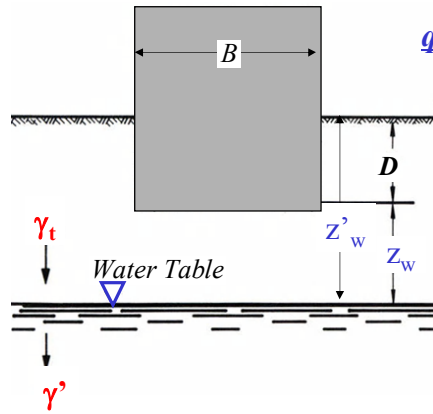
Area reduction factors for eccentrically loaded footings

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### 4.1.3 Effect of water table in sandy soil - effective stress -



$q_s$  in the second term of the BC equation

$$\begin{aligned} z'_w < D : q_s &\Rightarrow z'_w \gamma_t + (D - z'_w) \gamma' \\ D < z_w : q_s &\Rightarrow D \gamma_t \end{aligned} \quad (18)$$

$\gamma$  in the last term of the BC equation

$$\begin{aligned} z_w < 0 : \gamma &\Rightarrow \gamma' \\ 0 \leq z_w \leq B : \gamma &\Rightarrow \gamma' + \frac{z_w}{B} (\gamma_t - \gamma') \\ B \leq z_w : \gamma &\Rightarrow \gamma_t \end{aligned} \quad (19)$$

$$q_{ult} = Q_{ult} / B = cN_c + q_s N_q + \frac{\gamma B}{2} N_\gamma$$

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### 4.1.4 Effect of compressibility

dense  $\Rightarrow$  general failure

loose  $\Rightarrow$  local failure, punching shear failure  $\Rightarrow$  *settlement is more important than  $q_{ult}$  for loose soils.*

$\phi' > \psi$ : The looser, the smaller  $q_{ult}$ .

$\Rightarrow$  reduction of  $\phi'$  value

$$\phi \text{ in the BC eq. } \phi'_m \Rightarrow \tan^{-1} \left( \frac{2}{3} \tan \phi' \right) \quad (20) \quad \text{by Terzaghi}$$

bearing capacity factors for loose soils:  $N'_c, N'_q, N'_\gamma$   
(Table 3.2, p160, Das text book)

Modified  $f$  value by Davis (1968)

$$\tan \phi'_m = \frac{\cos \psi \sin \phi'}{1 - \sin \psi \sin \phi'} \quad (21)$$

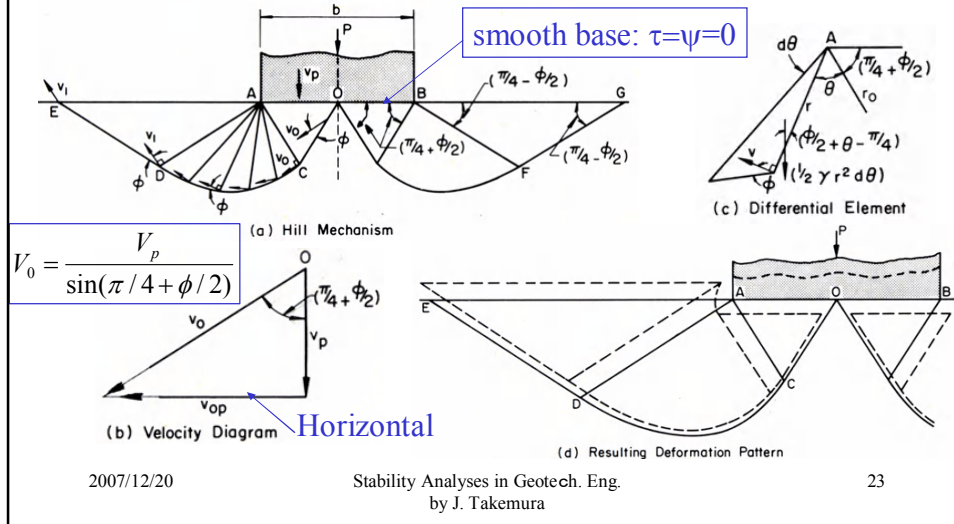
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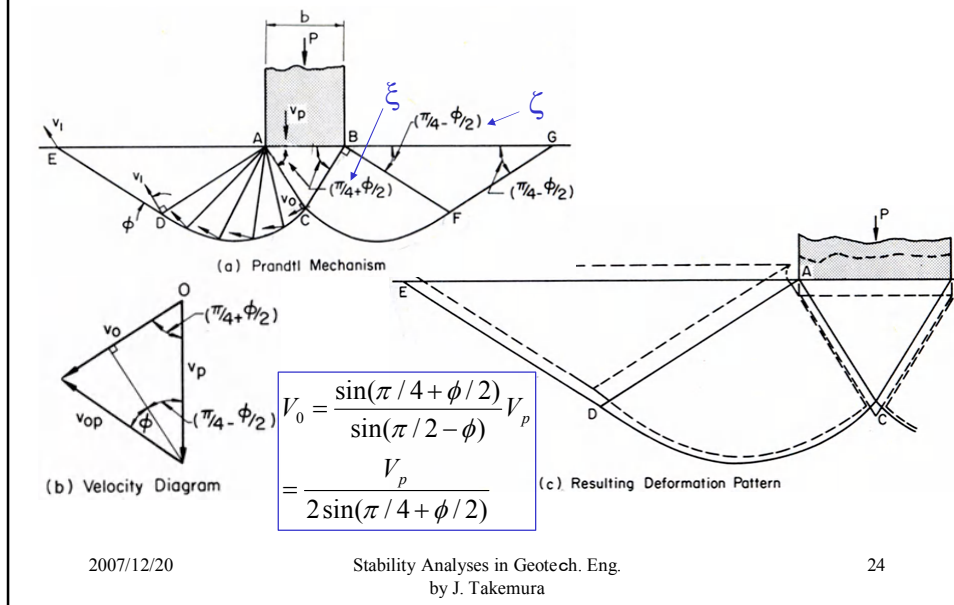
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### 4.1.4 Effect of footing base roughness

**Hill Mechanism** used for bearing capacity calculation of rigid footing with **smooth** base “Limit analysis and soil plasticity” Chen, (1975)



**Prandtl Mechanism** used for bearing capacity calculation of rigid footing with **rough** base “Limit analysis and soil plasticity” Chen, (1975)



•  $N_\gamma$  for smooth footing obtained from Hill Mechanism

$$[N_\gamma]_{Hill} = \frac{1}{4} \tan\left(\frac{1}{4}\pi + \frac{1}{2}\phi'\right) \left[ \tan\left(\frac{1}{4}\pi + \frac{1}{2}\phi'\right) \exp\left(\frac{3}{2}\pi \tan\phi'\right) - 1 \right] \quad (22)$$

$$+ \frac{3 \sin\phi'}{1 + 8 \sin^2\phi'} \left\{ \left[ \tan\left(\frac{1}{4}\pi + \frac{1}{2}\phi'\right) - \frac{\cot\phi'}{3} \right] \exp\left(\frac{3}{2}\pi \tan\phi'\right) + \tan\left(\frac{1}{4}\pi + \frac{1}{2}\phi'\right) \frac{\cot\phi'}{3} + 1 \right\}$$

$$[N_\gamma]_{Hill}: 0.72 (\phi'=10^\circ), 3.45 (\phi'=20^\circ), 15.2 (\phi'=30^\circ), 81.8 (\phi'=40^\circ)$$

•  $N_\gamma$  for rough footing obtained from Prandtl Mechanism

$$[N_\gamma]_{Prandtl} = 2[N_\gamma]_{Hill} \quad \text{for } \xi = 45^\circ + \phi'/2 \text{ and } \zeta = 45^\circ - \phi'/2 \quad (23)$$

The least  $[N_\gamma]_{Prandtl}$  is given by  $\frac{\partial [N_\gamma]_{Prandtl}}{\partial \xi} = 0, \frac{\partial [N_\gamma]_{Prandtl}}{\partial \zeta} = 0$

$$\text{ex) } [N_\gamma]_{Prandtl} = 26.6 \text{ for } \phi'=30^\circ \text{ when } \xi = 46^\circ \text{ and } \zeta = 45^\circ - \phi'/2 = 30^\circ$$

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## 4.2 Settlement of Foundation

### 4.2.1 Stress due to loading

Stresses due to a concentrated load (Boussinesq equation) on  
linear elastic halfspace

$$\Delta p = \Delta \sigma_z = \frac{3Pz^3}{2\pi L^5} = \frac{3P}{2\pi} \frac{z^3}{[r^2 + z^2]^{5/2}} \quad (24)$$

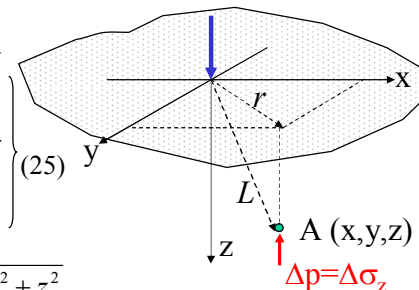
*vertical stress is  
independent on Poisson's  
ratio  $\nu$  and Young's  
modulus  $E$*

$$\Delta \sigma_x = \frac{P}{2\pi} \left\{ \frac{3x^2z}{L^5} - (1-2\nu) \left[ \frac{x^2 - y^2}{Lr^2(L+z)} + \frac{y^2z}{L^3r^2} \right] \right\}$$

$$\Delta \sigma_y = \frac{P}{2\pi} \left\{ \frac{3x^2z}{L^5} - (1-2\nu) \left[ \frac{y^2 - x^2}{Lr^2(L+z)} + \frac{x^2z}{L^3r^2} \right] \right\} \quad (25)$$

$$\Delta \tau_{rz} = \frac{3Qrz^2}{2\pi r^5}$$

$$r = \sqrt{x^2 + y^2} \quad L = \sqrt{x^2 + y^2 + z^2} = \sqrt{r^2 + z^2}$$



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## Vertical stress due to surface loading with uniform pressure $\Delta q_0$

integration of eq.(24)

### circular loading (center)

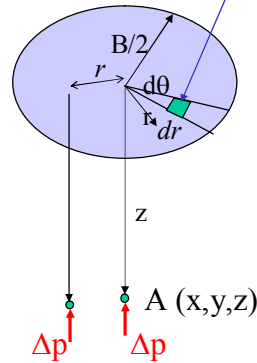
$$dp = \frac{3(\Delta q_0 r d\theta dr)}{2\pi z^2 \left[ 1 + \left( \frac{r}{z} \right)^2 \right]^{5/2}} \quad (25)$$

$$\Delta p = \int dp = \int_{\theta=0}^{\theta=2\pi} \int_{r=0}^{r=B/2} \frac{3(\Delta q_0 r d\theta dr)}{2\pi z^2 \left[ 1 + \left( \frac{r}{z} \right)^2 \right]^{5/2}}$$

$$= \Delta q_0 \left\{ 1 - \frac{1}{\left[ 1 + \left( \frac{B/2}{z} \right)^2 \right]^{3/2}} \right\} \quad (26)$$

*influential factor: I*

point load acting on small area



(Table 4.1, p222, Das text book)  
as function of  $z/(B/2)$  and  $r/(B/2)$

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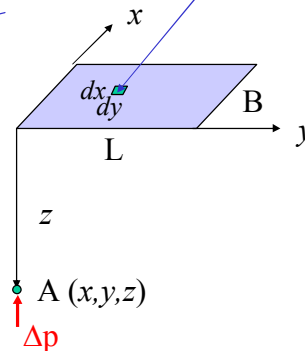
### Rectangular loading (at corner)

$$dp = \frac{3\Delta q_0 (dx dy) z^3}{2\pi [x^2 + y^2 + z^2]^{5/2}} \quad (27)$$

$$\Delta p = \int dp = \int_{y=0}^L \int_{x=0}^B \frac{3\Delta q_0 (dx dy) z^3}{2\pi [x^2 + y^2 + z^2]^{5/2}} = \Delta q_0 I \quad (28)$$

*influence factor  $I=f(m=B/z, n=L/z)$*

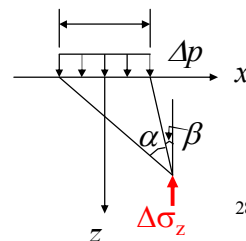
point load acting on small area



(Table 4.2, p224, 225,  
Figure 4.4, p226, Das text book)

### Strip loading footing

$$\Delta \sigma_z = \frac{\Delta q_0}{\pi} \{ \alpha + \sin \alpha (\alpha + 2\beta) \} \quad (29)$$



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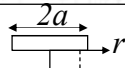
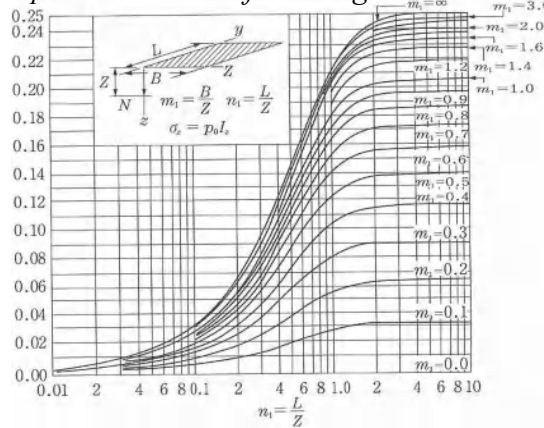
## Influence factors flexible circular and rectangular footing (at corner)

Circular  $F_c$

Influence  $F$

Corner of Rectangular  $F_r$

$z/a$	$r/a$					
	0	0.2	0.4	0.6	0.8	1.0
0	1.000	1.000	1.000	1.000	1.000	1.000
0.1	0.999	0.999	0.998	0.996	0.976	0.484
0.2	0.992	0.991	0.987	0.970	0.890	0.468
0.3	0.976	0.973	0.963	0.922	0.793	0.451
0.4	0.949	0.943	0.920	0.860	0.712	0.435
0.5	0.911	0.902	0.869	0.796	0.646	0.417
0.6	0.864	0.852	0.814	0.732	0.591	0.400
0.7	0.811	0.798	0.756	0.674	0.545	0.367
0.8	0.756	0.743	0.699	0.619	0.504	0.366
0.9	0.701	0.688	0.644	0.570	0.467	0.348
1.0	0.646	0.633	0.591	0.525	0.434	0.332
1.2	0.546	0.535	0.501	0.447	0.377	0.300
1.5	0.424	0.416	0.392	0.355	0.308	0.256
2.0	0.286	0.286	0.268	0.248	0.224	0.196
2.5	0.200	0.197	0.191	0.180	0.167	0.151
3.0	0.146	0.145	0.141	0.135	0.127	0.118
4.0	0.087	0.086	0.085	0.082	0.080	0.075

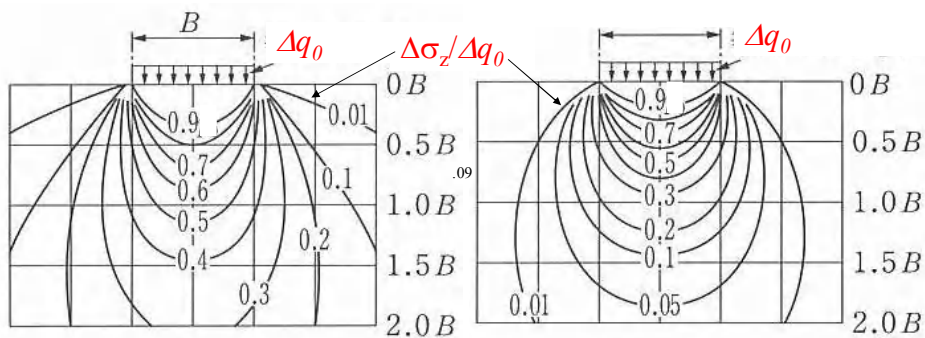


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## Contour of vertical stress $\sigma_z$ in semi-infinite elastic body subjected to uniform surface loading — Stress bulb (応力球根)—



Strip: 2D

Kusakabe "Soil Mechanics" (2004)

Circular: 3D

Deeper propagation

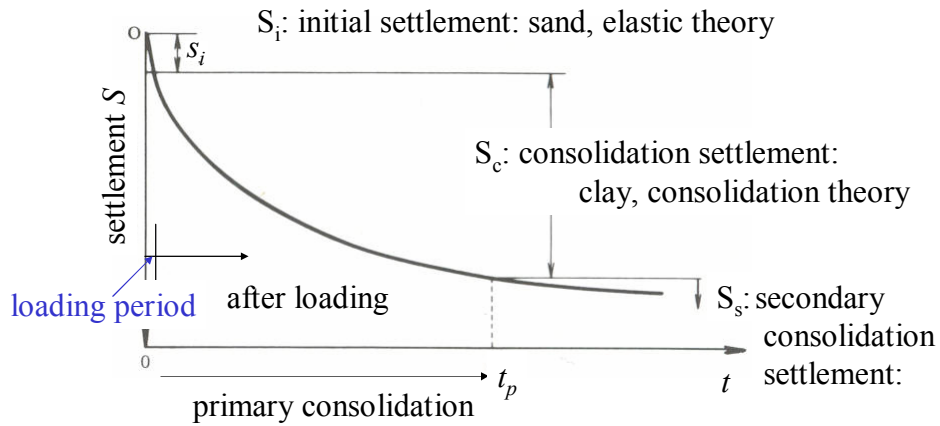
Shallower propagation  
 $\sim 0$  at  $z=2B$

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## 4.2.2 Settlement calculation



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## Elastic settlement by elasticity theory

for isotropic elastic media, by Hooke's law

$$S_e = \int_{z=0}^H \varepsilon_z dz = \frac{1}{E_s} \int_0^H (\Delta p_z - \nu_s \Delta p_x - \nu_s \Delta p_y) dz \quad (30)$$

$$S_e = \Delta q_0 B \frac{1 - \nu_s^2}{E_s} I_\rho \quad (31)$$

$\Delta q_0$ : net pressure applied from foundation

$B$ : width of the footing

$\nu_s$ : Poisson's ratio of soil

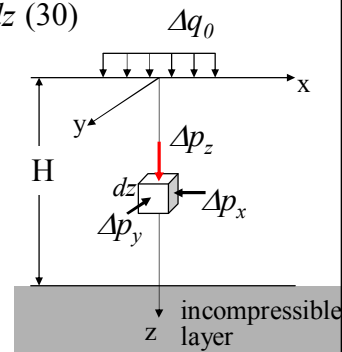
$E_s$ : Young's modulus of soil

$I_\rho$ : non-dimensional influence factor

rigid footing => uniform settlement

flexible footing (uniform loading)

=> non-uniform settlement



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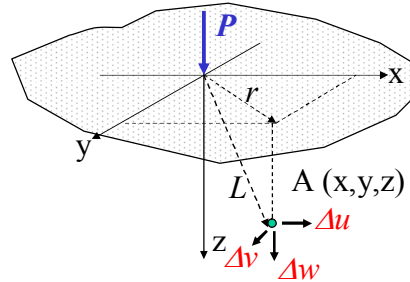


## Settlement caused by surface load

Boussinesq's equation :

Displacements due to a concentrated load (P) on linear elastic halfspace (u,v,w)

$$\left. \begin{aligned} \Delta w &= \frac{P(1+\nu)}{2\pi E} \left\{ \frac{z^2}{L^3} + \frac{2(1-\nu)}{L} \right\} \\ \Delta u &= \frac{P(1+\nu)}{2\pi E} \left\{ \frac{z}{L^3} - \frac{(1-\nu)}{L(L+z)} \right\} x \\ \Delta v &= \frac{P(1+\nu)}{2\pi E} \left\{ \frac{z}{L^3} - \frac{(1-\nu)}{L(L+z)} \right\} y \end{aligned} \right\} (32)$$



$$r = \sqrt{x^2 + y^2} \quad L = \sqrt{x^2 + y^2 + z^2} = \sqrt{r^2 + z^2}$$

Important => vertical displacement (w) at surface (z=0, L=r)

$$\Delta w = \frac{P(1-\nu^2)}{\pi E r} \quad (33)$$

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## Influence factors

Schleicher's solution (1926) on the influence factor  
for the corner of a flexible rectangular footing on infinite half space:  
(H=∞)

$$\alpha = \frac{1}{\pi} \left[ \ln \left( \frac{\sqrt{m^2+1}+m}{\sqrt{m^2+1}-m} \right) + m \ln \left( \frac{\sqrt{m^2+1}+1}{\sqrt{m^2+1}-1} \right) \right] \quad \text{where } m=L/B \quad (34)$$

$$S_e = \Delta q_0 B \frac{1-\nu_s^2}{E_s} \frac{\alpha}{2} \quad (35)$$

for the center of a flexible rectangular footing

$$S_e = \Delta q_0 B \frac{1-\nu_s^2}{E_s} \alpha \quad (36)$$

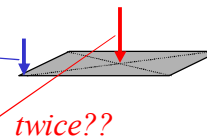
settlement of rigid rectangular footing

$$S_e = \Delta q_0 B \frac{1-\nu_s^2}{E_s} \alpha_r \quad (37)$$

uniform

(H=∞)

where m=L/B

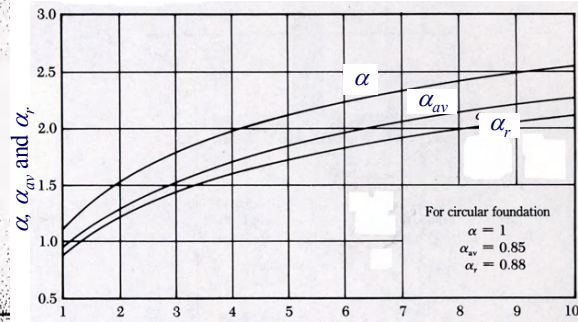
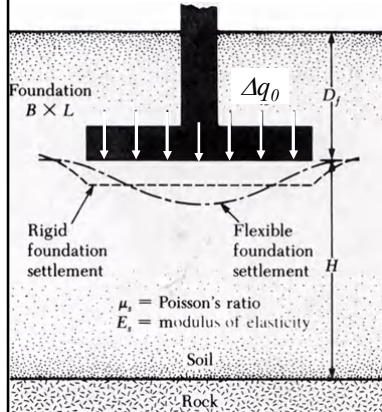


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## Elastic settlement of flexible and rigid foundations



"Principle of Foundation Engineering" B. M. Das (1999)

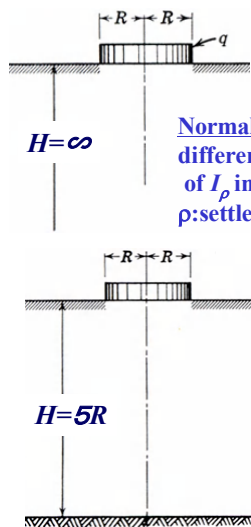
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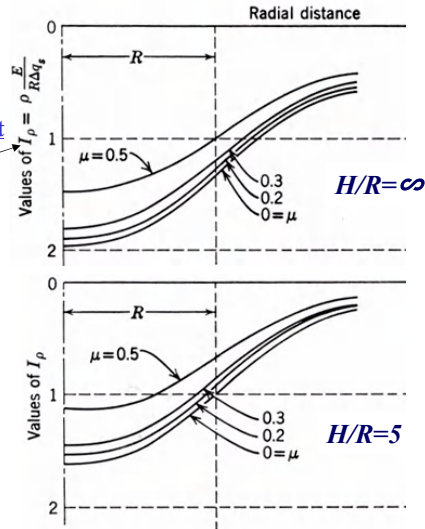
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## Effect of limited thickness of compressible layer -Influence factors under uniform circular load-

"Soil Mechanics" Lambe and Whitman



Normalized settlement  
different definition  
of  $I_p$  in eq.(31)  
 $\rho$ : settlement

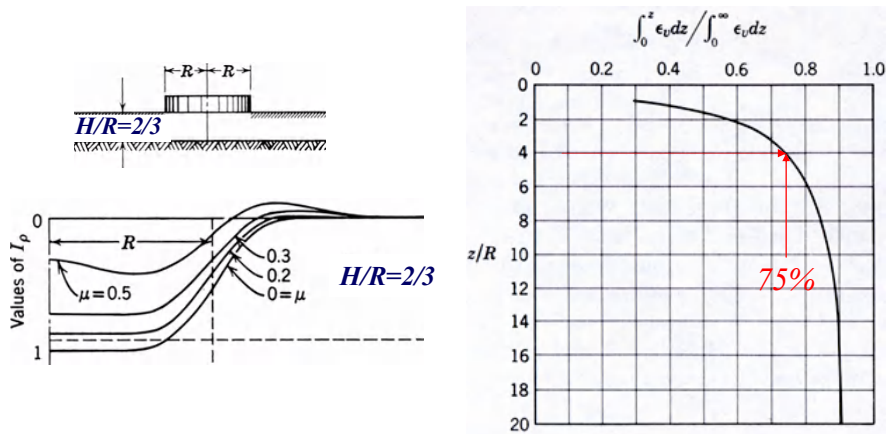


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## Effect of limited thickness of compressible layer



“Soil Mechanics” Lambe and Whitman

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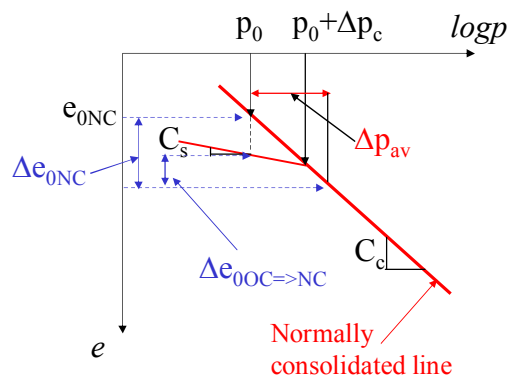
## Settlement due to consolidation

$$S_c = \int \varepsilon_z dz \quad (38)$$

for 1D condition

$$\varepsilon_z = \frac{\Delta e}{1 + e_0} \quad (39)$$

$\Delta p_{av}$ :  
average increase of pressure on  
the clay layer caused by  
loading from structure



$$\Delta p_{av} \approx \frac{1}{6} (\Delta p_{top} + 4\Delta p_{middle} + \Delta p_{bottom}) \quad (40)$$

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## Consolidation settlement calculation

$\varepsilon_z$  may be calculated from

$$\varepsilon_z = \frac{C_c}{1+e_0} \log\left(\frac{p_0 + \Delta p_{av}}{p_0}\right) \quad (41)$$

for NC

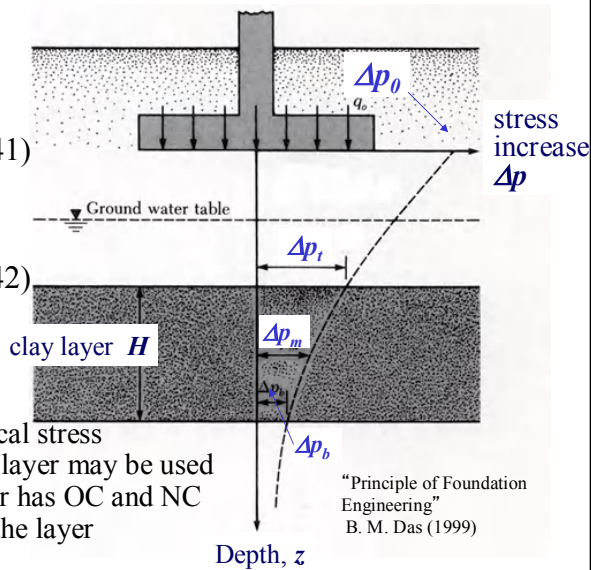
$$\varepsilon_z = \frac{C_r}{1+e_0} \log\left(\frac{p_0 + \Delta p_{av}}{p_0}\right) \quad (42)$$

for OC

$p_0$ :

average initial effective vertical stress

Stress at mid-depth of clay layer may be used as  $p_0$ . If clay layer is thick or has OC and NC portion, it is better to divide the layer into some sub-layers.



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## Field loading test

Estimation of settlement of Footing with  $B=B_F$   
using load settlement relation from Plate Load Test with  $B=B_P$

**for clayey soil:**

$$q_{ult(F)} = q_{ult(P)} \Rightarrow S_F = S_P \frac{B_F}{B_P} \quad (43)$$

*E: constant*

**for sandy soil:**

$$q_{ult(F)} = q_{ult(P)} \frac{B_F}{B_P} \Rightarrow S_F = S_P \left( \frac{2B_F}{B_F + B_P} \right)^2 \quad (44)$$

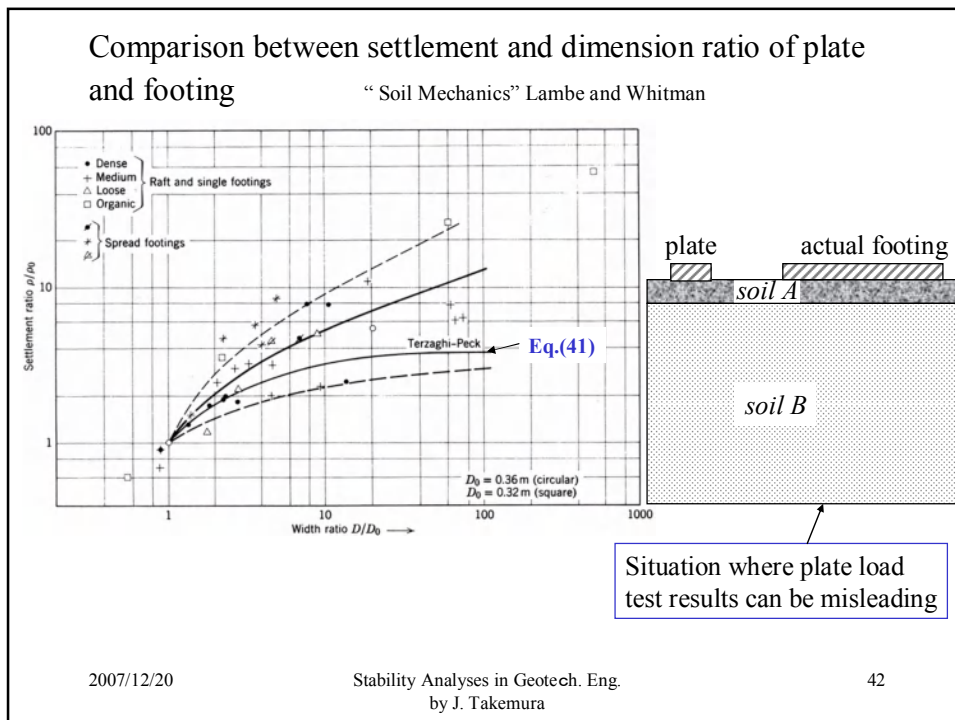
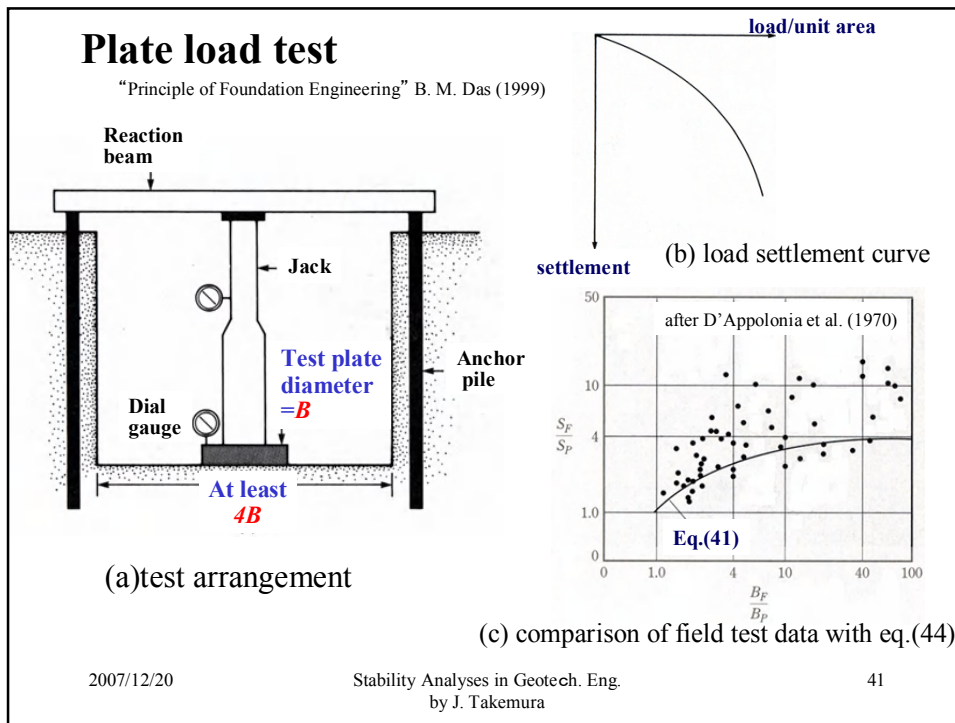
*empirical*

*E: increasing with depth but not linearly* Terzaghi and Peck(1967)

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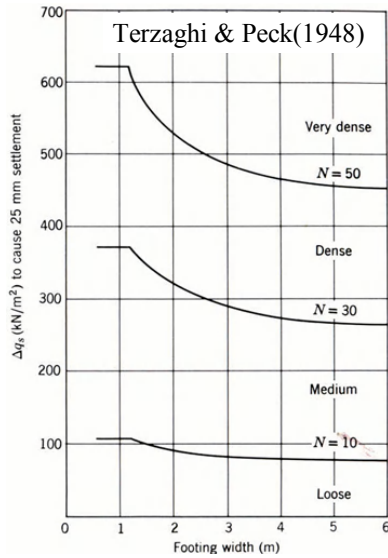
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## Prediction of settlement by penetration test

“Soil Mechanics” Lambe and Whitman




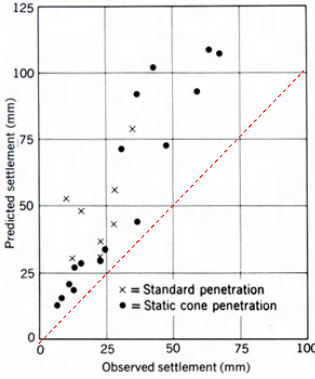
Meyerhof (1965)

$$\Delta q_s = 0.47 N S_F \quad B \leq 1.2m$$

$$\Delta q_s = 0.31 N S_F \left( \frac{B+0.3}{B} \right)^2 \quad B > 1.2m$$

$kN/m^2$     $mm$     $m$

over estimate  
  
 conservative  
 (safety side)



Settlement of footing from standard penetration test N

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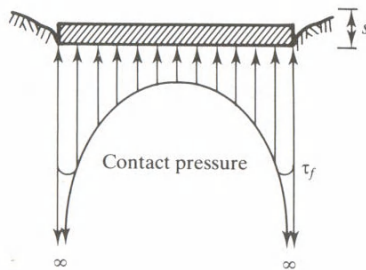
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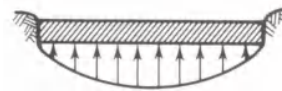
## Distribution of settlement and contact stress for rigid and flexible footing on cohesive ( $\phi=0$ ) and cohesionless ( $\phi>0$ ) material

cohesive ( $\phi=0$ ) material

“Soil Mechanics” Lambe and Whitman  
granular ( $\phi>0$ ) material



rigid footing



flexible footing



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